

1a) The  $t'$  axis coincides with Alan's worldline. The  $x'$  axis passes through  $t=x=0$  and has a slope of  $3/5$ .

Calibration is obtained with  $\gamma=5/4$ .

Thus, for  $t'=1$  ns, one has  $t=5/4$  ns = 1.25 ns.

For  $x' = 1$  ns one has  $x=5/4$  ns = 1.25 ns.

Event B is at the intersection between the  $x'$  axis and Brynna's worldline.

1b) You have to read the  $x$ -coordinates of events A and B on the diagram. To determine the time coordinate draw parallels to the  $x$  axis through events A and B. To determine the time coordinate draw parallels to the  $t$  axis through events A and B.

For event A one reads  $t_A=x_A=0$ .

For event B the spacetime diagram gives approximately  $t_B \approx 75$  ns and  $x_B \approx 125$  ns.

Thus, the displacement between the marks is about 125 ns in the tunnel frame, as read from the diagram.

1c) The Lorentz transformation equation gives

$$\Delta x = \gamma \Delta x' + \gamma v \Delta t'$$

$$\Delta t = \gamma \Delta t' + \gamma v \Delta x'$$

Using  $\Delta t' = 0$  and the Lorentz contraction to get  $\Delta x' = \gamma 80$  ns, one obtains

$$\Delta x = \gamma^2 80 \text{ ns} = 125 \text{ ns}$$

$$\Delta t = \gamma^2 v 80 \text{ ns} = v \Delta x = 75 \text{ ns}$$

Thus the graphical method agrees well with the result obtained using the algebraic method.

1d) The events A and B are simultaneous in the train frame. However, for an observer in the tunnel frame event A precedes event B. More precisely, given that event A (Alan paints a mark) happens at time  $t=0$  (and  $x=0$ ) in the tunnel frame, event B (Brynna paints a mark) happens at  $t=75$  ns, i.e. 75 ns after event A. In that amount of time Brynna has moved by  $\Delta x = (125-80)$  ns = 45 ns.

Brynna moving means that you follow her worldline on the spacetime diagram from the point  $x=80$  ns,  $t=0$  till event B.

2a) One can use the infinitesimal form of the Lorentz transformation equations.

We know  $v_x = 1/2$ . Since  $\beta$  is parallel to the  $x$ -axis, the velocity in the Other Frame will only have a non-zero  $x$  component.

The infinitesimal Lorentz transformation equations from the Home Frame to the Other Frame are

$$dx' = \gamma dx - \gamma \beta dt$$

$$dt' = \gamma dt - \gamma \beta dx$$

Dividing out and using  $v'_x = dx'/dt'$  and  $v_x = dx/dt$  one obtains

$$v'_x = (v_x - \beta)/(1 - \beta v_x) = -1/4$$

2b) Firstly, to answer that  $v$  remains 1 in both frames, you should explicitly show the invariance of  $v=1$  under the Einstein velocity transformations (we did that in class). Secondly, you can shortly provide the physical interpretation. The essence of the argument is as follows.

The existence of an invariant velocity  $v=1$  also implies that it is a true cosmic speed limit, in other words nothing can travel faster than the speed of light in vacuum  $v=1$ .

The invariance of  $v=1$  is related to the fact that if  $(\Delta s)^2=0$  in one inertial RF, then it is zero in all inertial RF. And  $(\Delta s)^2=0$  separates spacetime intervals  $(\Delta s)^2>0$  ( $v<1$ ) from spacetime intervals  $(\Delta s)^2<0$  ( $v>1$ ), thus avoiding violations of causality.

If  $v=1$  were not invariant, then we would violate causality. Why? Because then two events could be causally connected (i.e. connected by information moving with some velocity, not necessarily less than one) and at the same time have a spacetime interval  $(\Delta s)^2 < 0$ . But in this case one can always find an inertial RF where the two events are simultaneous, or even happen in reversed temporal order. This would mean a violation of causality, for which the cause must always precede the effect.

Other phrasing with similar logic could also be fine, obviously. And whatever drawing can be added—though the answer above would by itself ensure the full 2 points.